and I can only wonder that it has not had a more marked effect in popularising the general principle.

Prof. Thompson's illustrations from biology (attributed in part to Spencer) are, of course, of first-rate importance. Rayleigh.

The Age of the Earth.

Some fifty years ago Kelvin announced that the temperature of the earth could not have been anything like its present value for more than some 20–30 million years. This estimate was based upon three independent considerations, namely, the temperature gradient inside the earth's crust, the amount of tidal friction, and the total amount of energy radiated by the sun.

The first of these arguments has been invalidated completely by the discovery of the radio-active elements. The other two arguments are scarcely affected by this event.

The geologists always found some difficulty in compressing the history of the earth, more especially of the sedimentary strata, into the period allowed them by Kelvin. Prof. Harker's presidential address before the Yorkshire Geological Society, reprinted in your issue of March 25, seems to show that there is a general impression abroad that Kelvin's estimates have been superseded, and that the discoveries in radio-activity allow one to assume a period of the order of thousands of millions of years since the earth has reached a constant state as regards climate. I should like to be allowed to state as succinctly as possible what difficulties this view entails.

The mean temperature of the earth is about 280° absolute. It therefore radiates about $1{\cdot}7 \times 10^{24}$ ergs per second into space.

Assuming the latest value $1{\cdot}92 \frac{\text{cal}}{\text{cm.}^2\,\text{min.}}$ for the solar constant, the earth receives $1{\cdot}72 \times 10^{24}$ ergs per second from the sun. Therefore the radiation from the sun just compensates the amount lost by the earth; in other words, the temperature of the earth is determined by the temperature of the sun. The possibility that the earth's temperature might have been maintained by radio-active processes before the sun was incandescent, and that the radio-active substances have died off since then need scarcely be discussed seriously. For quite apart from the well-known sterilising effects of the rays, any radio-active substances with a sufficiently long life to keep up the temperature of the earth for any considerable length of time would not disappear quickly. Uranium, for instance, only diminishes at the rate of about 1 5 per cent. in 100 million years.

One may conclude, therefore, that the time during which the earth can have existed in its present state cannot be greater than the time since which the effective temperature of the sun has been about 6000°, its present value. This time cannot exceed about thirty million years. For the sun loses energy at the rate of about $3{\cdot}8 \times 10^{33} \frac{\text{ergs}}{\text{sec.}}$, and the total energy to be gained by a mass of $1{\cdot}97 \times 10^{33}$ gm., contracting to a radius $6{\cdot}96 \times 10^{10}$ cm., is $2{\cdot}2 \times 10^{48}$ ergs, assuming approximate homogeneity. (Taking the increase in density towards the centre into account does not alter these figures much.)' Now even if one assumes that the whole of this energy was radiated at a rate of about $3{\cdot}8 \times 10^{33} \frac{\text{erg}}{\text{sec.}}$, *i.e.*, at the present rate, it will only last 18·3 million years. But any other supposition, namely, that the sun at one time emitted more or less energy per second, leads to a shorter period for the earth in its present state.

To explain a greater age it was necessary to find other sources of energy, and since neither the heat of chemical combination nor any possible increase in the specific heat was anything like large enough, the heat of radio-active transformations was invoked. This was perhaps excusable in the early days before very much was known about the laws governing these processes, but it seems quite inadmissible to-day.

It has been suggested that at the enormous pressure and temperature inside the sun radio-active processes might be modified, and even that ordinary elements might break up. A consideration of the quantitative relations involved shows that this is most unlikely. Though one can scarcely apply ordinary thermodynamics to radio-active processes one can certainly apply the general rule, which may also be developed from the quantum theory if desired, namely, that a reaction the energy of which is A ergs per molecule is affected chiefly by the collisions of atoms of energy of the order A. Now A is of the order 10^{-5} ergs in radio-active processes, and one can therefore only expect the temperature to affect those if an appreciable number of atoms have an amount of energy of this order. The average energy of an atom would be 10^{-5} ergs at about $5{\cdot}10^{10}$ degrees. Therefore even at 500 million degrees only one atom amongst 10^{40} would be moving fast enough to influence a reaction which liberates 10^{-5} ergs. Obviously 500 million degrees is quite beyond the bounds of possibility in any part of the sun. One must conclude, therefore, that any process which liberates anything like the requisite energy is unaffected by solar conditions, and takes place at the same rate on the sun as on the earth. Thus one must fall back upon the ordinary radio-active materials, and as Sir Ernest Rutherford has pointed out, one would only gain a paltry five million years even if the whole sun were composed of uranium. The only way out would seem to be to suppose that the sun was created some 10^9 or 10^{10} years ago out of special radio-active material which produces an enormous amount of energy, and that it has been breaking up ever since. This material does not exist on the earth though, so the earth would have to be the object of a special creation. Such an assumption, of course, can neither be controverted nor even discussed. But unless some such hypothesis is introduced, *i.e.*, unless the presumably radio-active solar material which liberates a quantity of energy sufficient to keep up the sun's heat for the desired 10^9 or 10^{10} years, is supposed to have been created by some inconceivable force at the epoch at which the sun is supposed to have begun to radiate, this material would have disintegrated long ago. It might be objected that the same holds good of uranium, that the fact that uranium exists in measurable quantities proves that it has not existed for a time great in comparison to $5{\cdot}10^9$ years.

This is doubtless true, but there is no real difficulty about assuming uranium or other radio-active substances to have been produced if one supposes the solar system to have been formed by the collision of two stars.

At the moment of collision the velocity of two stars half the mass of the sun would be $\frac{1{\cdot}15 \times 10^{13}}{\sqrt{r}} \frac{\text{cm.}}{\text{sec.}}$, r being the distance between the centres of gravity. Suppose they both contained some lead, this would reach a temperature of the order $\frac{1{\cdot}1 \times 10^{20}}{r}$, *i.e.*, of the order $2{\cdot}10^9$ degrees at the moment of collision. As has been shown above, an appreciable quantity of radio-active

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material might be formed at such a temperature if some helium were present.

But of course the heat used up in forming these substances would cool the rest of the mass: any energy gained in radio-active form would be lost in the form of heat. It could never avail to explain a solar constant such as has been measured for longer than Kelvin's 20 million years. In other words, radioactive substances produced would act only as accumulators of energy, not as primary batteries.

To recapitulate: As Kelvin showed, gravitational energy can only account for 18·3 million years of sunshine at the present rate. Invoking radio-activity as a source of energy implies the assumption that unknown radio-active materials liberating considerably more energy than uranium were created by some unknown agency within a measurable period of time, and that these are now breaking up. This assumption is not necessary to account for the existence of uranium, as it is quite conceivable that a certain amount of radio-active matter might be produced afresh during every stellar collision. The energy of substances formed in this way would not be available to explain a greater amount of energy on the sun as their energy is abstracted from the gravitational energy, and has already been taken into account.

F. A. Lindemann.

Sidholme, Sidmouth, April 5.

Harmonic Analysis.

In a paper which I read to the Physical Society last January (see Nature, February 11, p. 662) I suggested that the best way of analysing a wave, the graph of which was given, was to apply the rules for the mechanical quadrature of integrals which are given in treatises in the calculus of finite differences. I am convinced that these methods when applied intelligently are much simpler and ever so much more accurate than most, if not all, of the methods in everyday use.

In the paper referred to above I applied a well-known method of mechanical quadrature (Weddle's rule) to the case of a semicircular alternating wave, the equation to the positive half of which is $y=\sqrt{x-x^2}$. I chose this wave because I found that the evaluation of the Fourier integrals for it by analysis was laborious. Prof. A. E. Kennelly, of Harvard University, has kindly written to me to point out that the equation to the curve can be readily put in the form—

$$y=J_1(\pi/2)\sin\pi x-(1/3)J_1(3\pi/2)\sin 3\pi x+ \\ (1/5)J_1(5\pi/2)\sin 5\pi x- \ldots$$

where $J_1(x)$ is the Bessel's function of the first order. Hence from tables of these functions we get:—

$$y=0{\cdot}567\sin\pi x+0{\cdot}0939\sin 3\pi x. \\ +0{\cdot}0422\sin 5\pi x+0{\cdot}0252\sin 7\pi x. \\ +0{\cdot}0171\sin 9\pi x+ \ldots,$$

Very close approximations to these numbers can be obtained very simply by Weddle's rule. For example, if b_1 denote the amplitude of the first harmonic, we have:

$$10b_1=5y_{1/6}+\sqrt{3}v_{1/3}+6y_{1/2},$$

where $y_n=\sqrt{n-n^2}$, and hence $b_1=0{\cdot}568$.

To get an accuracy of the same order for the third, fifth, seventh, and ninth harmonics we must calculate or measure the lengths of 8, 13, 18, and 23 ordinates respectively. Doing this, we find that $b_3=0{\cdot}0942$, $b_5=0{\cdot}0423$, and that b_7 and b_9 are given correctly. It will be seen that from the practical point of view the simplicity and accuracy of the method in this case leave little to be desired. It has the great advantage that the amplitude of each harmonic can be computed independently of the others.

When the wave passes smoothly through the extremities of the ordinates we measure, we can apply the rule with confidence. Jagged or very distorted waves must be treated more carefully. For example, if we apply the rule to a rectangular alternating wave of height unity we find from the formula given above that $10b_1=11+\sqrt{3}$, and so $b_1=1{\cdot}27321$ approx. The true value is $4/\pi$, *i.e.*, 1·27324 . . ., and hence the error is less than 1 in 40,000. For a triangular alternating wave of height unity, however, if we apply the rule intelligently we get $b_1=0{\cdot}88$ . . . instead of 0·81057 . . . The error in this case arises from applying Weddle's rule through a point of discontinuity. If we apply it over one-quarter of the wave, it being necessary to measure six ordinates instead of three, we find that $b_1=0{\cdot}81056$ . . .

Alexander Russell.

Faraday House, Southampton Row, W.C.,
April 12.

A Mistaken Butterfly.

Referring to Prof. Barnard's letter so titled in Nature of April 15, which describes the apparent mistake of a butterfly in visiting a peacock's feather as if expecting to "extract food," I think it probable that there are no animals that do not make mistakes at times. I observed an analogous mistake made by a species of Pieridæ—*Appias nero*—in Sumatra, as I have recorded in "A Naturalist's Wanderings," p. 130:—"In the open paths I netted scarlet Pieridæ . . . often flying in flocks of over a score, exactly matching in colour the fallen [withered] leaves, which it was amusing to observe how often they mistook for one of their own fellows at rest, and to watch the futile attentions of an amorous male towards such a leaf moving in the wind."

Henry O. Forbes.

Redcliffe, Beaconsfield, Bucks,
April 17.

The "Green Ray" at Sunset.

Prof. A. W. Porter, in Nature of February 18 (vol. xciv., p. 672), seems to think that the "green ray" is more of a subjective phenomenon than anything else, or at least often is so; but the fact that it is seen at sunrise also shows that in this case at least it is not a result of complementary colours. Besides, if it were a subjective phenomenon, one would expect to see it on every occasion when the sun set behind a clear horizon, whereas the sight is somewhat rarer. I once saw a lovely blue flash, and I read a description recently of a sunset in Palestine where the writer speaks of the sun vanishing like a blue spark. If you hold a lens almost edgeways on between your eye and a light and move it until it is quite edgeways on a few discs of light will be seen, and at last these vanish in a green or blue flash, the effect of dispersion.

35 Roeland Street, Cape Town,
March 17.

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